# MA114 Summer 2018
# Worksheet 26 – Differential Eq. I – 7/30/18

1. a) Is $y = \sin(3x) + 2e^{4x}$ a solution to the differential equation $y'' + 9y = 50e^{4x}$? Explain why or why not.
   b) Explain why every solution of $dy/dx = y^2 + 6$ must be an increasing function.
   c) What does it mean to say that a differential equation is linear or nonlinear?

2. Find all values of $\alpha$ so that $y(x) = e^{\alpha x}$ is a solution of the differential equation $y'' + y' - 12y = 0$.

3. Show that for any value of $c$, $y = x^2 + \dfrac{c}{x^2}$ is a solution to the differential equation $xy' + 2y = 4x^2, (x > 0)$. Find the value of $c$ for which the solution satisfies the initial condition $y(6) = 5$.

4. Sketch the slope field of the differential equation. Then use it to sketch a solution curve that passes through the given point.
   a) $y' = y - 2x, (1, 0)$
   b) $y' = xy - x^2, (0, 1)$

5. Consider the autonomous (only $y$ and derivatives appear) differential equation $y' = y^2(3 - y)(y + 1)$.
   a) Find the equilibrium values of $y$.
   b) Sketch the phase portrait (direction field) of this differential equation, paying particular attention to the behavior near equilibria.
   c) Classify each equilibrium point as *stable* or *unstable*.

6. Consider the autonomous differential equation from the previous problem. Without solving the differential equation, determine the value of $\lim_{y\to\infty} y(t)$, where the initial value is
   a) $y(0) = 1$
   b) $y(0) = 4$
   c) $y(0) = -4$